Mesh generation for fusion applications

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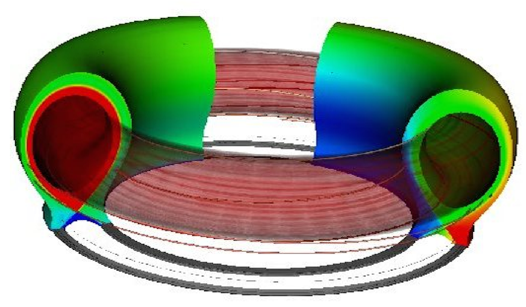
Context and motivations

- High anisotropy in magnetized fusion plasmas:
  ⇒ requires the use of **flux aligned meshes**.
- Complex and realistic geometries:
  ⇒ need other strategies (equidistribution, orthogonality).
- High order derivatives (in MHD for example):
  ⇒ require **regular representation**.



Sketch of a Tokamak reactor

Unified code for the generation of flux aligned mesh in the **poloidal plane**.

- Different codes and type of meshes.
- Different numerical methods:
  - Semi-Lagrangian approaches.
  - Finite difference, Finite volume
  - Finite Element (Spline or Hermite-Bezier on quadrangles; Powell-Sabin, Clough-Tocher on triangles, etc..)

Workflow diagram

Input data:
Flux field ψ from equilibrium code + the original mesh
(+ boundary definition of the vacuum vessel)

→ Data regularization

Segmentation:
Decomposition of the domain into several patches
→ 1 patch is homeomorphic to a logical square

Splines approximation:
Construction of mesh grids of individual patches

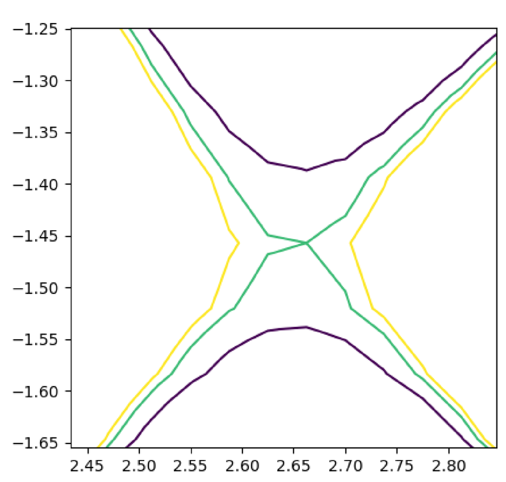
↔ Wall interaction treatment

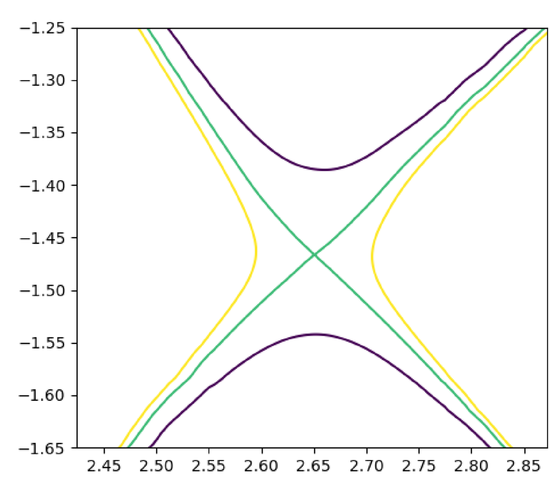
Gluing of diffrent patches to obtain the global mesh
⇒ export the description into HDF5 output

Pre-processing of the data

Isolines regularization

If the equilibrium solver results are not smooth enough : low-order discretization, coarse resolution, ..
⇒ Replace the flux function by its Clough Tocher interpolant on the refined mesh (locally).

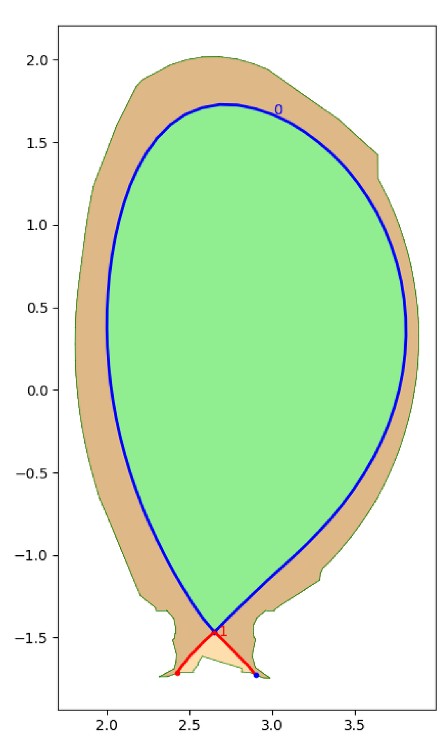


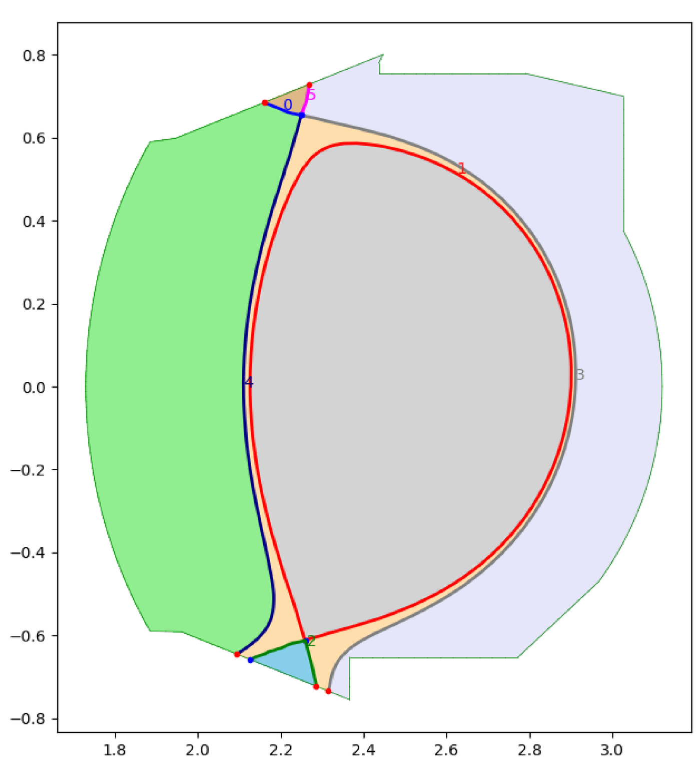


Segmentation

- Based on Morse theory and Reeb graph.
- Automatic decomposition of isolines including arbitrary number of X-Points.
- Topological set of the isolines consists of finite connected components and contains only closed orbits.

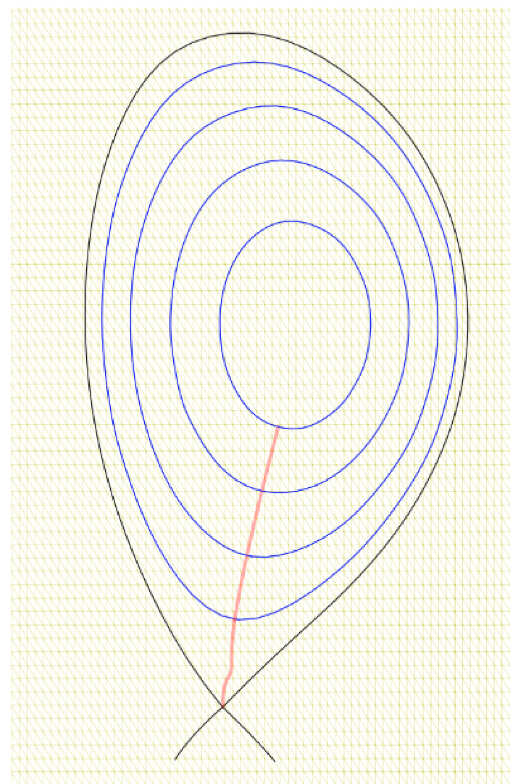
Examples of domain segementation for JET and WEST tokamaks:





Structured grid construction

I. Isolines generation

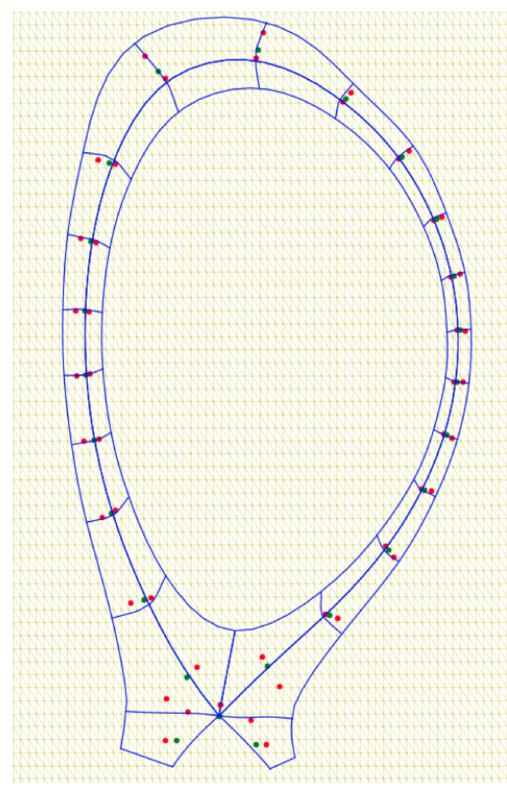


1. Identify a mapping: $(s,t) \in [0,1] \times [0,1] \rightarrow \mathbf{x}(s,t) \in \Omega$
2. Get the radial curve $\mathscr{S}(s)$, solution of the ODE:
$$\begin{cases} \dfrac{d\mathbf{x}}{ds} = \dfrac{\psi_M - \psi_0}{\|\nabla\psi\|^2}\nabla\psi \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases}$$
3. Compute the spline interpolation of the radial curve and generate a set of internal isolines.

II. Meshing of individual patch Ω_k

1. Choose a finite number of isolines $f^{-1}(c_i)$.
2. Approximate each level set by a spline: $f^{-1}(c_i) \sim \mathscr{C}_i(t) = \sum_j \mathbf{C}_j^i N_j(t)$.
   N_j: Bspline Basis.
   $\mathbf{C}_j^i$: Control Points.
3. Construct a 2D tensor product mapping $[0,1] \times [0,1] \rightarrow \Omega_k$:
$$\mathcal{S}(s,t) = \sum_i \sum_j \mathbf{P}_{i,j} N_i(s) N_j(t), \text{ s.t. } \mathcal{S}(s_i,t) = \mathscr{C}_i(t), \forall i.$$

III. Gluing patches

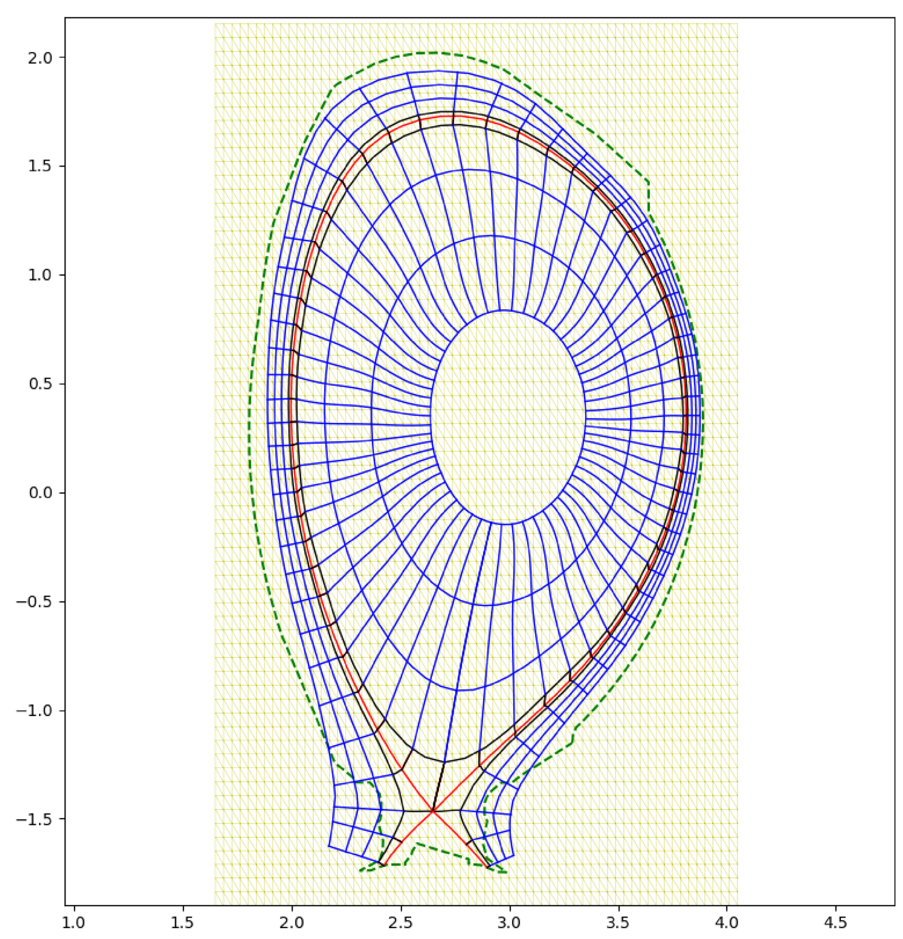


Given 2 subdomains Ω_1 and Ω_2 described by:
$$\mathcal{S}_1(s,t) = \sum_i \sum_j \mathbf{P}_{i,j} N_i(s) N_j(t)$$
$$\mathcal{S}_2(s,t) = \sum_i \sum_j \mathbf{Q}_{i,j} N_i(s) N_j(t)$$
with a common boundary for instance: $\mathcal{S}_1(0,t) = \mathcal{S}_2(0,t)$

1. $\mathcal{C}^0$ continuity: $\mathbf{P}_{0,j} = \mathbf{Q}_{0,j}, \forall j$.
2. $\mathcal{G}^1$ continuity: geometric condition on the control points, $\mathbf{P}_{1,j}, \mathbf{P}_{0,j} = \mathbf{Q}_{0,j}, \mathbf{Q}_{1,j}$ have to be aligned.

Multipatch example

Flux aligned mesh for JET (Core/Edge/SOL regions):



Summary

Development of a mesh generation software for tokamak simulations:

- Can be used by different codes and handle different types of meshes.
- Written in Python with Fortran bindings:
  → using open source libraries (except Segmentation module but free for academics).

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